Reflected entropy and Markov gap in non-inertial frame

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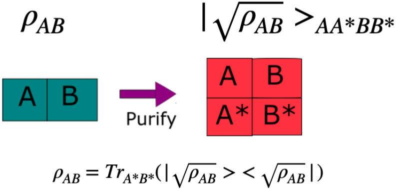
Motivation

- Entanglement entropy is enough for bipartite pure state.
- What about bipartite mixed state?
- Any trace of multipartite entanglement!
- If yes! How does it change in non-inertial frame?

[Dutta, Faulkner : 19]

- *Purification:* A bipartite quantum system $A \cup B$ in a mixed state ρ_{AB} is prepared by embedding the system $A \cup B$ in a larger tripartite system $A \cup B \cup C$.
- Reflected entropy $S_R(A:B)$ of a bipartite system AB involves the canonical purification of the given mixed state ρ_{AB} by doubling its Hilbert space to define a pure state $|\sqrt{\rho_{AB}}\rangle_{ABA^*B^*}$ such that $\rho_{AB} = \mathrm{Tr}_{A^*B^*} |\sqrt{\rho_{AB}}\rangle \langle\sqrt{\rho_{AB}}|$.



- Reflected entropy is defined as

$$S_R(A:B) = S(AA^*).$$

- The reflected entropy satisfy the bound,

$$\min\{2S_A, 2S_B\} \geq S_R(A:B) \geq I(A:B).$$

Markov gap

- The Markov gap is defined as [Hayden, Parrikar and Sorce: 2021]
$$h(A:B) = S_R(A:B) - I(A:B).$$
- Following the lower bound of reflected entropy, $h(A,B) \geq 0$.
- It is a measure of tripartite entanglement. [Akers and Rath: 2019, Zou et al: 2021]
- The Markov gap can be written as conditional mutual information [Dutta and Faulkner: 2019]
$$h(A:B) = I\left(A:B* \mid B\right)$$
where, $I(A:B* \mid B) = I(A:BB*) - I(A:B)$

The setup

- First case: we consider two non-inertial observers Alice and Bob sharing an initially entangled bipartite fermionic field mode described by the Bell state

$$|B\rangle_{AB} = \alpha\,|0\rangle_A\,|0\rangle_B + \sqrt{1-\alpha^2}\,|1\rangle_A\,|1\rangle_B\,, \quad \alpha \in (0,1)$$

- Second case: we consider a tripartite entangled fermionic field mode described by the GHZ state

$$|GHZ\rangle_{ABC} = \alpha\,|0\rangle_A\,|0\rangle_B\,|0\rangle_C + \sqrt{1-\alpha^2}\,|1\rangle_A\,|1\rangle_B\,|1\rangle_C\,, \quad \alpha \in (0,1)$$

- Third case: we consider a tripartite entangled fermionic field mode described by the W state

$$|W\rangle_{ABC} = \alpha\,|1\rangle_A\,|0\rangle_B\,|0\rangle_C + \alpha\,|0\rangle_A\,|0\rangle_B\,|1\rangle_C + \sqrt{1-2\alpha^2}\,|0\rangle_A\,|1\rangle_B\,|0\rangle_C\,, \quad \alpha \in (0, \frac{1}{\sqrt{2}})$$

- For all the cases, Bob moves with uniform acceleration

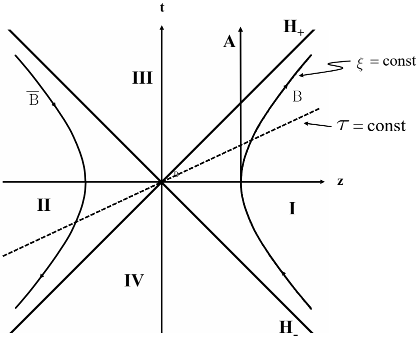
Rindler diagram

- Assume the observer Bob is accelerating in $(1+1)$-d Minkowski spacetime
$$ds^2 = -dt^2 + dz^2$$

- Rindler coordinates (τ, ξ) are appropriate to describe an observer moving with uniform acceleration in Minkowski spacetime.

- Two sets of Rindler coordinates are required as,
$$t = a^{-1} e^{a\xi} \sinh a\tau, \qquad z = a^{-1} e^{a\xi} \cosh a\tau, \quad \text{Region I},$$
$$t = -a^{-1} e^{a\xi} \sinh a\tau, \quad z = -a^{-1} e^{a\xi} \cosh a\tau, \quad \text{Region II},$$
where a is Bob's proper acceleration.

- The metric in Rindler coordinates is
$$ds^2 = e^{2a\xi} \left(-d\tau^2 + d\xi^2 \right)$$



Rindler spacetime diagram. Source: Alsing at., el 2006

- $\xi = const$ are hyperbolas and $\tau = const$ are lines through the origin.
- Region I and II are causally disconnected.

- Minkowski states for Bob is related to the Rindler states through the Bogoliubov transformations

$$|0\rangle_B = \cos r\,|0\rangle_B\,|0\rangle_{\bar{B}} + \sin r\,|1\rangle_B\,|1\rangle_{\bar{B}}\,,$$
$$|1\rangle_B = |1\rangle_B\,|0\rangle_{\bar{B}}\,.$$

  where $r = \tan^{-1}\exp\left(-\frac{\pi\omega}{a}\right)$ is the acceleration parameter and ω is the mode frequency.

- Note that we have adopted the single mode approximation, which means Bob's detector is sensitive to a single mode frequency.

- **Bell state:**

$$|B\rangle_{AB\bar{B}} = \alpha \cos r|000\rangle_{AB\bar{B}} + \alpha \sin r|011\rangle_{AB\bar{B}} + \sqrt{1-\alpha^2}|110\rangle_{AB\bar{B}},$$

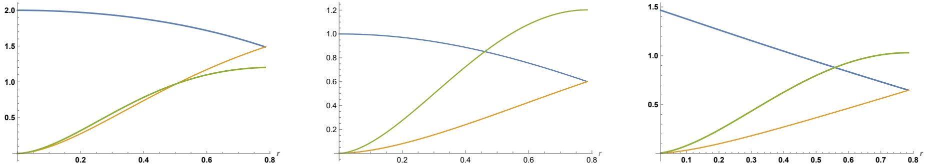
- **GHZ state:**

$$|GHZ\rangle_{AB\bar{B}C} = \alpha \cos r|0000\rangle_{AB\bar{B}C} + \alpha \sin r|0110\rangle_{AB\bar{B}C} + \sqrt{1-\alpha^2}|1101\rangle_{AB\bar{B}C},$$

- **W state:**

$$\begin{aligned}|W\rangle_{AB\bar{B}C} = \alpha \cos r|1000\rangle_{AB\bar{B}C} + \alpha \sin r|1110\rangle_{AB\bar{B}C} + \alpha \cos r|0001\rangle_{AB\bar{B}C} \\ + \alpha \sin r|0111\rangle_{AB\bar{B}C} + \sqrt{1-2\alpha^2}|0100\rangle_{AB\bar{B}C}.\end{aligned}$$

- Performing canonical purification we obtain ρ_{ABA*B*} and therefore ρ_{AA*}.



(a) Bell state: $S_R(A:B)$ (Blue), $S_R(A:\bar{B})$ (orange), and $S_R(B:\bar{B})$ (green). Here, we have taken $\alpha = \frac{1}{\sqrt{2}}$.

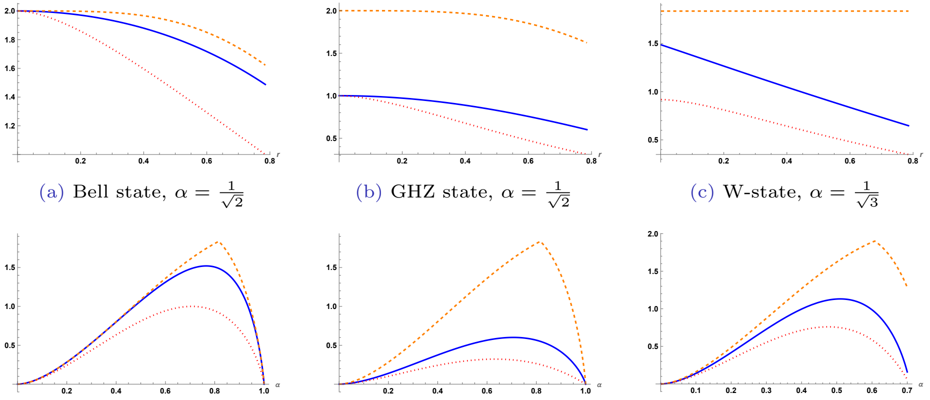
(b) GHZ state: $S_R(A:B)$ (Blue), $S_R(A:\bar{B})$ (orange), and $S_R(B:\bar{B})$ (green). Here, we have taken $\alpha = \frac{1}{\sqrt{2}}$.

(c) W-state: $S_R(A:B)$ (Blue), $S_R(A:\bar{B})$ (orange), and $S_R(B:\bar{B})$ (green). Here, we have taken $\alpha = \frac{1}{\sqrt{3}}$.

Figure 1: S_R *vs* r.

- Reflected entropy is degraded with increasing acceleration
- Reflected entropy is finite at infinite acceleration
- $S_R(A:B) = S_R(A:\bar{B})$ at $r = \pi/4$

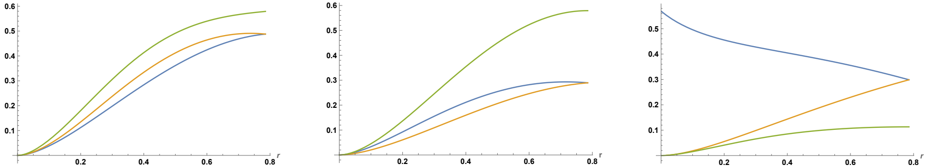
Reflected entropy bound



(a) Bell state, $\alpha = \frac{1}{\sqrt{2}}$ (b) GHZ state, $\alpha = \frac{1}{\sqrt{2}}$ (c) W-state, $\alpha = \frac{1}{\sqrt{3}}$

(d) Bell state, $r = \frac{\pi}{4}$ (e) GHZ state, $r = \frac{\pi}{4}$ (f) W-state, $r = \frac{\pi}{8}$

Figure 2: $S_R(A:B)$ (blue), $min\{2S_A, 2S_B\}$ (orange), $I(A:B)$ (red) are plotted wrt r and α.



(a) Bell state: $h(A:B)$ (blue), $h(A:\bar{B})$ (orange) and $h(B:\bar{B})$ (green). Here, $\alpha = \frac{1}{\sqrt{2}}$.

(b) GHZ state: $h(A:B)$ (blue), $h(A:\bar{B})$ (orange) and $h(B:\bar{B})$ (green). Here, $\alpha = \frac{1}{\sqrt{2}}$.

(c) W-state: $h(A:B)$ (blue), $h(A:\bar{B})$ (orange) and $h(B:\bar{B})$ (green). Here, $\alpha = \frac{1}{\sqrt{3}}$.

Figure 3: h *vs* r.

- Tripartite entanglement in present in Bell state.
- Tripartite entanglement increase in Bell and GHZ state
- Tripartite entanglement decreases in W state
- $h(A:B) = h(A:\bar{B})$ at $r = \pi/4$

Problems with reflected entropy

- **Monotonicity:** If δ_ρ is a measure of correlations, then it must satisfy the inequality,
$$\delta_\rho(A:BC) \geq \delta_\rho(A:B)$$
i.e, it must be monotonically decreasing under partial trace.
- Very recently it has been argued, for any $\xi \in (0,2)$ there exists a density operator ρ_{ABC} on $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C = \mathbb{C}^3 \otimes \mathbb{C}^3 \otimes \mathbb{C}^2$ for which the ξ-th Rényi reflected entropy satisfies, [Hayden, Lemm, Sorce: 2023]
$$S_R^\xi(A:BC) \leq S_R^\xi(A:B)$$
- The argument can be generalized for $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C = \mathbb{C}^{n+1} \otimes \mathbb{C}^{m+1} \otimes \mathbb{C}^2$

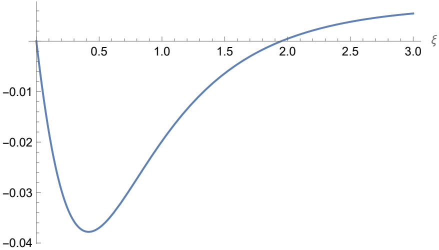
Problems with reflected entropy

- Consider the state,

$$\rho_{ABC} = \frac{1}{2na + 2(m-1)b}\Big[a|000\rangle\langle 000| + a|110\rangle\langle 110| + \sum_{m,n}\Big(a|n00\rangle\langle n00| + a|n10\rangle\langle n10|) + b|0m0\rangle\langle 0m0| + b|1m1\rangle\langle 1m1|\Big)\Big]$$

where, $n, m \geq 2$.

- $S_R^{\xi}(A:BC) - S_R^{\xi}(A:B)$ shows,



$n, m = 2,\ a = 200,\ b = 10$

- These results heavily depend on the values of a and b.

Summary

- We have computed Reflected entropy for bipartite and tripartite fermionic field modes described by Bell, GHZ and W state respectively.
- Reflected entropy decreases between Alice and Bob due to Unruh effect, and in the infinite acceleration limit it reaches a non-vanishing final value.
- We have showed that Markov gap which was introduced as a tripartite entanglement measure changes monotonically with acceleration.
- We have checked the monotonicity of Renyi reflected entropy for 3-qubit or 4-qubit state where it shows to be monotonic. For other dimensions (qudit-qudit-qubit) there are some exceptions.

Future directions

- check the same for bosonic degrees of freedoms in non intertial frame
- black hole background analysis
- understand the non-monotonicity of reflected entropy and Markov gap
- consider other states for better understanding of multipartite entanglement
- Others !!!

Thank You!